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Equivalent bosonic theory for the massive Thirring model with a non-local interaction

Kang Li†‡§ and Carlos Naón†

† Departmento de Física, Universidad Nacional de La Plata, CC 67, 1900 La Plata, Argentina

‡ Department of Physics, Hangzhou University, Hangzhou, 310028, People's Republic of China

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Abstract. We study, through path-integral methods, an extension of the massive Thirring model in which the interaction between currents is non-local. By examining the mass expansion of the partition function we show that this non-local massive Thirring model is equivalent to a certain non-local extension of the sine-Gordon theory. Thus, we establish a non-local generalization of the well known Coleman's equivalence. We also discuss some possible applications of this result in the context of one-dimensional strongly correlated systems and finite-size quantum field theories.

1. Introduction

Bosonization, i.e. the equivalence between fermionic and bosonic Green functions in $1+1$ dimensions, has a long history [1], that could be traced back to the work of Bloch [2] on the energy loss of charged particles travelling through a metal. In more recent years, the well known Coleman's equivalence proof between the massive Thirring and sine-Gordon theories [3] and Polyakov and Wiegman and Witten's non-Abelian bosonization [4], helped us to convert this procedure into a standard and powerful tool for the understanding of quantum field theories (QFTs). All these achievements were realized in the context of *local* QFTs.

Recently, the bosonization procedure in its path-integral version [5] was applied, for the first time, to a *non-local* QFT, namely, a Thirring model with massless fermions and a non-local (and non-covariant) interaction between fermionic currents [6]. The study of such a model is relevant, not only from a purely field-theoretical point of view but also because of its connection with the physics of strongly correlated systems in one spatial dimension (1d). Indeed, this model describes an ensemble of non-relativistic particles coupled through a two-body forward-scattering potential and displays the so-called Luttinger-liquid behaviour [7] that could play a role in real 1d semiconductors (see for instance [8]).

In this paper we undertake the path-integral bosonization of the non-local Thirring model (NLT) with a relativistic fermion mass term included in the action. Using a functional decoupling technique to treat the non-locality [6], and performing a perturbative expansion in the mass parameter, we find that the NLT is equivalent to a purely bosonic action which is a simple non-local extension of the sine-Gordon model. Thus, our main result can be considered as a generalization of Coleman's equivalence to the case in which the usual Thirring interaction is point-split through bilocal potentials. In the language of many-body,

§ E-mail address: naon@venus.fisica.unlp.edu.ar

non-relativistic systems, the relativistic mass term can be shown to represent not an actual mass, but the introduction of backward-scattering effects [9]. Therefore, our result provides an alternative route to explore the dynamics of collective modes in 1d strongly correlated systems.

This paper is organized as follows. In section 2 we present the model and write the partition function in terms of a massive fermionic determinant. Then we perform a perturbative expansion in the mass parameter and evaluate every free (massless) vacuum expectation value (VEV). This allows us to obtain an explicit expression for the partition function of the massive NLT. In section 3 we introduce a modified non-local sine-Gordon model (NLSG) with an additional non-local term. We then consider the corresponding partition function, make an expansion in the cosine term and compute the free VEV's, for each term in the series. Finally we compare both the fermionic and bosonic expansions term by term and find that they are equal if a certain relationship between NLT and NLSG potentials is satisfied. We end section 3 by briefly showing how our approach can be used to study the 1d electronic liquid with back-scattering. We also comment on the possibility of exploiting this work in order to shed some light onto the validity of Coleman's equivalence at finite volume. In the final section we summarize and stress the main aspects of this work.

2. Partition function for the massive non-local Thirring model

Let us consider the Euclidean Lagrangian density of the massive Thirring model with a non-local interaction between fermionic currents

$$L = \mathrm{i}\bar{\Psi}\partial\!\!\!/\Psi + \tfrac{1}{2}g^2 \int \mathrm{d}^2 y\, J_\mu(x) V_{(\mu)}(x, y) J_\mu(y) - m\bar{\Psi}\Psi \tag{2.1}$$

where

$$J_\mu(x) = \bar{\Psi}(x)\gamma_\mu \Psi(x) \tag{2.2}$$

and

$$V_{(\mu)}(x, y) = V_{(\mu)}(|x - y|) \tag{2.3}$$

is an arbitrary function of $|x - y|$. We shall use γ matrices defined as

$$\gamma_0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \gamma_1 = \begin{pmatrix} 0 & \mathrm{i} \\ -\mathrm{i} & 0 \end{pmatrix} \qquad \gamma_5 = \mathrm{i}\gamma_0\gamma_1 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{2.4}$$

$$[\gamma_\mu, \gamma_\nu] = 2\delta_{\mu\nu} \qquad \gamma_\mu\gamma_5 = \mathrm{i}\epsilon_{\mu\nu}\gamma_\nu. \tag{2.5}$$

The partition function of the model is

$$Z = N \int \mathrm{D}\bar{\Psi}\, \mathrm{D}\Psi\, \mathrm{e}^{-\int \mathrm{d}^2 x\, L}. \tag{2.6}$$

By using the following representation of the functional delta,

$$\delta(C_\mu) = \int \mathrm{D}\tilde{A}_\mu\, \exp\left(- \int \mathrm{d}^2 x\, \tilde{A}_\mu C_\mu \right) \tag{2.7}$$

we can write Z as

$$Z = N \int \mathrm{D}\bar{\Psi}\, \mathrm{D}\Psi\, \mathrm{D}\tilde{A}_\mu\, \mathrm{D}\tilde{B}_\mu \times \exp\left\{ - \int \mathrm{d}^2 x \left[\bar{\Psi}(\mathrm{i}\partial\!\!\!/ - m)\Psi + \tilde{A}_\mu \tilde{B}_\mu + \frac{g}{\sqrt{2}}(\tilde{A}_\mu J_\mu + \tilde{B}_\mu K_\mu) \right] \right\} \tag{2.8}$$

where

$$K_\mu(x) = \int d^2y\, V_{(\mu)}(x, y) J_\mu(y). \tag{2.9}$$

Please note that no sum over repeated indices is implied when a subindex (μ) is involved.

If we define

$$\bar{B}_\mu(x) = \int d^2y\, V_{(\mu)}(y, x)\tilde{B}_\mu(y) \tag{2.10}$$

$$\tilde{B}_\mu(x) = \int d^2y\, b_{(\mu)}(y, x)\bar{B}_\mu(y) \tag{2.11}$$

with $b_{(\mu)}(y, x)$ satisfying

$$\int d^2y\, b_{(\mu)}(y, x)V_{(\mu)}(z, y) = \delta^2(x - z) \tag{2.12}$$

and change auxiliary variables in the form

$$A_\mu = \frac{1}{\sqrt{2}}(\tilde{A}_\mu + \bar{B}_\mu) \tag{2.13}$$

$$B_\mu = \frac{1}{\sqrt{2}}(\tilde{A}_\mu - \bar{B}_\mu) \tag{2.14}$$

we obtain

$$Z = N \int DA_\mu\, DB_\mu\, \det(i\partial\!\!\!/ + gA\!\!\!/ - m)e^{-S(A,B)} \tag{2.15}$$

where

$$S(A, B) = \tfrac{1}{2} \int d^2x \int d^2y\, b_{(\mu)}(x, y)[A_\mu(x)A_\mu(y) - B_\mu(x)B_\mu(y)]. \tag{2.16}$$

The Jacobian associated with the change $(\tilde{A}, \tilde{B}) \to (A, B)$ is field independent and can be absorbed in the normalization constant N.

We have been able to express Z in terms of a fermionic determinant. This fact will enable us to apply the machinery of the path-integral approach to bosonization, first developed in the context of local theories, to the present non-local case. However, before we do this some remarks are in order. First, it is worth noting that, as a consequence of the change of bosonic variables (equations (2.13) and (2.14)), the effect of the non-local interaction has been completely transferred to the purely bosonic piece of the action, $S[A, B]$. On the other hand we see that the B-field is completely decoupled from both the A-field and the fermion field. Keeping this in mind, it is instructive to try to recover the partition function corresponding to the usual covariant Thirring model ($b_{(0)}(y, x) = b_{(1)}(x, y) = \delta^2(x - y)$), starting from (2.15). In doing so one readily discovers that B_μ describes a negative-metric state whose contribution must be factorized and absorbed in N in order to get a sensible answer for Z. This procedure parallels, in the path-integral framework, the operator approach of Klaiber [10], which precludes the use of an indefinite-metric Hilbert space. If one follows the same prescription in the present non-local case, the result is:

$$Z = N \int DA_\mu\, \det(i\partial\!\!\!/ + gA\!\!\!/ - m)e^{-S[A]} \tag{2.17}$$

with

$$S(A) = \tfrac{1}{2} \int d^2x\, d^2y\, b_{(\mu)}(x, y)A_\mu(x)A_\mu(y). \tag{2.18}$$

As it is known, the massive determinant in (2.17) has not been exactly solved yet. The usual way of dealing with it consists of performing a chiral transformation in the fermionic path-integral variables and then making an expansion with m as its perturbative parameter. (This procedure was employed for the local case in [5].) Since we were able to write the partition function in such a way that non-local terms are not present in the fermionic determinant, we can follow exactly the same strategy as in the local case. To this aim let us first express the vector field in terms of two new fields Φ and η as

$$A_\mu(x) = -\epsilon_{\mu\nu}\partial_\nu\Phi(x) + \partial_\mu\eta(x) \tag{2.19}$$

which can be considered as a change of bosonic variables with trivial (field-independent) Jacobian. We also make the change

$$\Psi(x) = \exp[-g(\gamma_5\Phi(x) + \mathrm{i}\eta(x))]\chi(x) \tag{2.20}$$

$$\bar\Psi(x) = \bar\chi(x)\exp[-g(\gamma_5\Phi(x) - \mathrm{i}\eta(x))]. \tag{2.21}$$

The Jacobian corresponding to the above transformations is non-trivial due to the so-called chiral anomaly. Its detailed evaluation has been given several times in the literature [11]. The result is

$$J_F = \exp\left[\frac{g^2}{2\pi}\int \mathrm{d}^2x\ \Phi(x)\,\Box\,\Phi(x)\right]. \tag{2.22}$$

We then get

$$Z = N\int \mathrm{D}\bar\chi\ \mathrm{D}\chi\ \mathrm{D}\Phi\ \mathrm{D}\eta\mathrm{e}^{-S_{\mathrm{eff}}} \tag{2.23}$$

where

$$S_{\mathrm{eff}} = S_{0F} + S_{0\mathrm{B}} - m\int \mathrm{d}^2x\ \bar\chi\mathrm{e}^{-2g\gamma_5\Phi}\chi \tag{2.24}$$

$$S_{0F} = \int \mathrm{d}^2x\ (\bar\chi\mathrm{i}\partial\!\!\!/\chi) \tag{2.25}$$

and

$$\begin{aligned} S_{0\mathrm{NLB}} = \frac{g^2}{2\pi}\int \mathrm{d}^2x\ (\partial_\mu\Phi)^2 &+ \frac{1}{2}\int \mathrm{d}^2x\ \mathrm{d}^2y\ \epsilon_{\mu\lambda}\epsilon_{\mu\sigma}b_{(\mu)}(y,x)\partial_\lambda\Phi(x)\partial_\sigma\Phi(y) \\ &+\tfrac{1}{2}\int \mathrm{d}^2x\ \mathrm{d}^2y\ b_{(\mu)}(y,x)\partial_\mu\eta(x)\partial_\mu\eta(y) \\ &-\int \mathrm{d}^2x\ \mathrm{d}^2y\ [b_{(0)}(y,x)\partial_0\eta(x)\partial_1\Phi(y) - b_{(1)}(y,x)\partial_1\eta(x)\partial_0\Phi(y)]. \end{aligned} \tag{2.26}$$

Note that for

$$\partial_1^x\partial_0^y b_0(x,y) = \partial_0^x\partial_1^y b_1(x,y) \tag{2.27}$$

the last term of $S_{0\mathrm{NLB}}$ vanishes, and in this case $\eta(x)$ decouples from $\bar\chi$, χ and Φ. In the general case $S_{0\mathrm{NLB}}$ describes a system of two bosonic fields coupled by distance-dependent coefficients.

Exactly as one does in the local case, the partition function for the massive NLT can be formally written as a mass expansion:

$$Z = \sum_{n=0}^{\infty}\frac{(m)^n}{n!}\left\langle\prod_{j=1}^{n}\int \mathrm{d}^2x_j\ \bar\chi(x_j)\mathrm{e}^{-2g\gamma_5\Phi(x_j)}\chi(x_j)\right\rangle_0 \tag{2.28}$$

where the $\langle\ \rangle_0$ means VEV in the theory of free massless fermions and non-local bosons. Using the identity

$$\bar{\chi}(x_j)\mathrm{e}^{-2g\gamma_5\Phi(x_j)}\chi(x_j) = \mathrm{e}^{-2g\Phi}\bar{\chi}\frac{1+\gamma_5}{2}\chi + \mathrm{e}^{2g\Phi}\bar{\chi}\frac{1-\gamma_5}{2}\chi \tag{2.29}$$

equation (2.28) can be written as

$$Z = \sum_{k=0}^{\infty}\frac{(m)^{2k}}{k!^2}\int\prod_{i=1}^{k}\mathrm{d}^2x_i\,\mathrm{d}^2y_i\left\langle\exp\left[2g\sum_i(\Phi(x_i)-\Phi(y_i))\right]\right\rangle_{0\mathrm{B}} \\ \times\left\langle\prod_{i=1}^{k}\bar{\chi}(x_i)\frac{1+\gamma_5}{2}\chi(x_i)\bar{\chi}(y_i)\frac{1-\gamma_5}{2}\chi(y_i)\right\rangle_{0F}. \tag{2.30}$$

Each fermionic part can be readily computed by writing

$$\begin{aligned}\bar{\chi}\frac{1+\gamma_5}{2}\chi &= \bar{\chi}_1\chi_1\\ \bar{\chi}\frac{1-\gamma_5}{2}\chi &= \bar{\chi}_2\chi_2\end{aligned} \tag{2.31}$$

where $\chi = \binom{\chi_1}{\chi_2}$, $\bar{\chi} = (\bar{\chi}_1, \bar{\chi}_2)$, and using Wick's theorem with the usual free fermion propagator.

In view of the bosonic (non-local) factors, they are more easily handled in momentum space. The corresponding Fourier transformed action acquires the following more compact form:

$$S_{0\mathrm{B}} = \frac{1}{(2\pi)^2}\int\mathrm{d}^2p\,\{\hat{\Phi}(p)\hat{\Phi}(-p)A(p) + \hat{\eta}(p)\hat{\eta}(-p)B(p) - \hat{\Phi}(p)\hat{\eta}(-p)C(p)\} \tag{2.32}$$

where

$$A(p) = \frac{g^2}{2\pi}p^2 + \frac{1}{2}[\hat{b}_{(0)}(p)p_1^2 + \hat{b}_{(1)}(p)p_0^2] \tag{2.33}$$

$$B(p) = \tfrac{1}{2}[\hat{b}_{(0)}(p)p_0^2 + \hat{b}_{(1)}(p)p_1^2] \tag{2.34}$$

$$C(p) = [\hat{b}_{(0)}(p) - \hat{b}_{(1)}(p)]p_0p_1 \tag{2.35}$$

$p^2 = p_0^2 + p_1^2$, and $\hat{\Phi}$, $\hat{\eta}$ and $\hat{b}_{(\mu)}$ are the Fourier transforms of Φ, η and $b_{(\mu)}$ respectively.

One then has

$$\left\langle\exp\left[2g\sum_i(\Phi(x_i)-\Phi(y_i))\right]\right\rangle_{0\mathrm{B}} = \frac{\int\mathrm{D}\hat{\Phi}(p)\,\mathrm{D}\hat{\eta}(p)\,\mathrm{e}^{-S_{0\mathrm{B}}}\cdot\mathrm{e}^{\frac{g}{\pi}\sum_i\int\mathrm{d}^2p\,\Phi(p)(\mathrm{e}^{\mathrm{i}px_i}-\mathrm{e}^{\mathrm{i}py_i})}}{\int\mathrm{D}\hat{\Phi}(p)\,\mathrm{D}\hat{\eta}(p)\mathrm{e}^{-S_{0\mathrm{B}}}}. \tag{2.36}$$

This VEV can be computed by translating the quantum fields $\hat{\Phi}(p)$ and $\hat{\eta}(p)$,

$$\begin{aligned}\hat{\Phi}(p) &= \hat{\phi}(p) + E(p)\\ \hat{\eta}(p) &= \hat{\rho}(p) + F(p)\end{aligned} \tag{2.37}$$

where $\hat{\phi}$ and $\hat{\rho}$ are the new quantum fields, whereas $E(p)$ and $F(p)$ are two classical functions satisfying

$$\begin{aligned}E(-p) &= -\frac{4gB(p)}{\Delta(p)}D(p,x_i,y_i)\\ F(-p) &= -\frac{2gC(p)}{\Delta(p)}D(p,x_i,y_i)\end{aligned} \tag{2.38}$$

with

$$\Delta = C^2(p) - 4A(p)B(p) \tag{2.39}$$

and

$$D(p, x_i, y_i) = \sum_i (e^{ipx_i} - e^{ipy_i}). \tag{2.40}$$

We then get

$$\left\langle \exp\left[2g\sum_i(\Phi(x_i) - \Phi(y_i))\right]\right\rangle_{0B} = \exp\left\{-\left(\frac{g}{\pi}\right)^2\int d^2p\,\frac{B(p)}{\Delta(p)}D(p, x_i, y_i)D(-p, x_i, y_i)\right\}. \tag{2.41}$$

Combining this result with the Fourier transformed fermionic factors [12], we find

$$Z = \sum_{k=0}^{\infty}\frac{(m)^{2k}}{(k!)^2}\int\prod_{i=1}^{k}d^2x_i\,d^2y_i \times \exp\left\{-\int\frac{d^2p}{(2\pi)^2}\left[\frac{2\pi}{p^2} - \frac{2\pi g^2(\hat{b}_{(0)}p_0^2 + \hat{b}_{(1)}p_1^2)}{g^2(\hat{b}_{(0)}p_0^2 + \hat{b}_{(1)}p_1^2)p^2 + \pi\hat{b}_{(0)}\hat{b}_{(1)}p^4}\right] \times D(p, x_i, y_i)D(-p, x_i, y_i)\right\}. \tag{2.42}$$

Thus, we have been able to obtain an explicit expansion for the partition function of a massive Thirring model with arbitrary (symmetric) bilocal potentials coupling the fermionic currents. This result will be used in the next section in order to establish, by comparison, its equivalence to a sine-Gordon-like model.

3. Connection with a non-local sine-Gordon model

Let us now consider the Lagrangian density of the NLSG given by

$$\mathcal{L}_{\rm NLSG} = \frac{1}{2}(\partial_\mu\phi(x))^2 + \frac{1}{2}\int d^2y\,\partial_\mu\phi(x)d_{(\mu)}(x-y)\partial_\mu\phi(y) - \frac{\alpha_0}{\beta^2}\cos\beta\phi \tag{3.1}$$

where $d_{(\mu)}(x-y)$ is an arbitrary potential function of $|x-y|$. The partition function of this model reads

$$Z_{\rm NLSG} = \int D\phi\,\exp\left[-\int d^2x\,\mathcal{L}_{\rm NLSG}\right]. \tag{3.2}$$

Performing a perturbative expansion in α_0, we obtain

$$Z_{\rm NLSG} = \sum_{k=0}^{\infty}\left[\frac{1}{k!}\right]^2\left(\frac{\alpha_0}{\beta^2}\right)^{2k}\int\prod_{i=1}^{k}d^2x_i\,d^2y_i\,\langle e^{i\beta\sum_i(\phi(x_i)-\phi(y_i))}\rangle_0 \tag{3.3}$$

where $\langle\,\rangle_0$ means the VEV with respect to the 'free' action defined by the two first terms in the right-hand side of equation (3.1). Since we are again led to the computation of VEVs of vertex operators, from now on the technical aspects of the calculation are, of course, very similar to those depicted in the previous section. For this reason we shall omit the details here. The result is

$$Z_{\mathrm{NLSG}} = \sum_{k=0}^{\infty} \left[\frac{1}{k!}\right]^2 \left(\frac{\alpha_0}{\beta^2}\right)^{2k} \int \prod_{i=1}^{k} \mathrm{d}^2 x_i \, \mathrm{d}^2 y_i \times \exp\left[-\frac{\beta^2}{4} \int \frac{\mathrm{d}^2 p}{(2\pi)^2} \frac{D(p, x_i, y_i) D(-p, x_i, y_i)}{\frac{1}{2} p^2 + \frac{1}{2}(\hat{d}_{(0)}(p) p_0^2 + \hat{d}_{(1)}(p) p_1^2)} \right]. \tag{3.4}$$

By comparing equation (2.42) with equation (3.4), we find that both expansions are identical if the following equations hold:

$$m = \frac{\alpha_0}{\beta^2} \tag{3.5}$$

and

$$\frac{1}{\frac{g^2}{\pi}\left(\frac{p_0^2}{\hat{b}_{(1)}} + \frac{p_1^2}{\hat{b}_{(0)}}\right) + p^2} = \frac{\beta^2}{4\pi(p^2 + \hat{d}_{(0)} p_0^2 + \hat{d}_{(1)} p_1^2)} \tag{3.6}$$

where, for the sake of clarity, we have omitted the p-dependence of the potentials.

Therefore, we have obtained a formal equivalence between the partition functions of the massive NLT and NLSG models. This is the main result of this paper. Let us stress that in the present path-integral framework we cannot establish a direct identification between fermionic and bosonic variables as it was done, for instance, in the fermionization of the 2d Ising model [13] or in Mandelstam's operational bosonization [14]. Thus, our result should be rigorously understood as an equivalence between Green functions. However, an operational analysis of the NLT model following the lines of [14] should lead to a nonlinear relationship between fermionic and bosonic fields, similar to the one obtained in the local case.

In order to check the validity of equation (3.6), let us specialize it to the covariant case,

$$\begin{aligned} \hat{b}_{(0)}(p) &= \hat{b}_{(1)}(p) = \hat{b}(p) \\ \hat{d}_{(0)}(p) &= \hat{d}_{(1)}(p) = \hat{d}(p) \end{aligned} \tag{3.7}$$

which yields

$$\frac{1}{\frac{g^2}{\pi \hat{b}(p)} + 1} = \frac{\beta^2}{4\pi(1 + \hat{d}(p))}. \tag{3.8}$$

In particular, when $\hat{b}(p) = 1$ and $\hat{d}(p) = 0$, our massive NLT model returns to the usual massive Thirring model, and the NLSG model becomes the ordinary sine-Gordon model. Making these replacements in equation (3.8) we get

$$\frac{\beta^2}{4\pi} = \frac{1}{1 + \frac{g^2}{\pi}} \tag{3.9}$$

which is the well known Coleman's result [3]. Of course, in this particular case one also has a modified version of the identity (3.5), with both m and α_0 renormalized due to divergencies coming from the vertex operators VEVs.

It is certainly encouraging to reproduce equation (3.9). However, our more general formula (3.6) enables us to profit from the bosonization identification in a much wider variety of situations, and in a very straightforward way. In particular, the non-local version of the sine-Gordon model can be easily used in the context of 1d strongly correlated fermions. This type of system has recently attracted much attention, due to striking advances in the material sciences that have allowed us to build real 'quantum wires' [15]. Much of the theoretical understanding of these physical systems has come from the study of the Tomonaga–Luttinger (TL) model [16–18] which, in its simpler version, describes spinless fermions interacting through their density fluctuations. In [6] it was shown that the TL model is a particular case of the NLT model considered in the present work, corresponding to $\hat{b}_{(1)} \to \infty$ and $\hat{b}_{(0)}$ associated to the density–density interaction, $V(p_1) = \frac{1}{\hat{b}_{(0)}(p)}$. For this many-body system, adding a relativistic fermion mass is intimately connected to the description of backward-scattering processes (the so-called Luther–Emery model [9]). Therefore, the NLSG model could be used to explore the Luther–Emery model. For illustrative purposes we shall consider here the spinless case, although the extension to the spin-$\frac{1}{2}$ case can be easily done within this framework (see [6]). To do this, according to the previous discussion, one has to take the limit $\hat{b}_{(1)} \to \infty$ in (3.6), thus obtaining

$$\frac{1}{\frac{g^2}{\pi} V(p_1) p_1^2 + p^2} = \frac{\beta^2}{4\pi(p^2 + \hat{d}_{(0)} p_0^2 + \hat{d}_{(1)} p_1^2)}. \tag{3.10}$$

In this context the above equation has to be viewed as an identity that permits us to determine the potentials $\hat{d}_\mu$ necessary to analyse the original fermionic Luther–Emery model in terms of the bosonic NLSG model. For instance, if we set $\hat{d}_{(0)} = 0$ and $\beta^2 = 4\pi$, $\hat{d}_{(1)}$ turns out to be proportional to V. Then equation (3.1) describes the dynamics of the collective modes, whose spectrum, as it is well known, develops a gap due to back-scattering effects. By virtue of equation (3.5) one can directly read the value of this gap from (3.1), obtaining $m = \alpha_0/4\pi$.

In passing let us mention that the study of the Luther–Emery model in the presence of impurities could also be undertaken by combining the present scheme with the results of [19].

We believe that the identification established in this paper might be useful to compute finite-size corrections in the massive Thirring model. There has been some recent interesting studies on this subject which found different results for the values of the central charges of the massive Thirring [20] and sine-Gordon [21] models. One possible explanation for this disagreement was given in [22], where it was argued, by using perturbed conformal field theory, that Coleman's equivalence is spoilt by finite-volume effects. All of these investigations are restricted to local models. In this context, our approach could be employed to examine the influence of non-contact interactions on the perturbed conformal properties of the systems. Since such a computation is expected to be closely related to the ground-state structure of the theories under consideration, it will be facilitated by recent results on the vacuum properties of the NLT model [23]. This problem is beyond the scope of this paper, but will be addressed in the near future.

4. Summary

In this work we have considered an extension of the massive Thirring model in which the fermionic current–current interaction is mediated by distance-dependent potentials. We also introduced a simple modification of the sine-Gordon model that consists of adding a non-local kinetic-like term to the usual bosonic action. By analysing the vacuum-to-vacuum functionals of each model through perturbative expansions, in complete analogy with the original procedure followed by Coleman in his well known paper [3], we found that both series (the mass expansion of the Thirring-like model and the 'fugacity' expansion of the sine-Gordon-like model) are equal provided that a certain relation between the corresponding potentials is satisfied. This is our main result (see equation (3.6)). Taking into account the close connection between the non-local Thirring model and a non-relativistic many-body system of one-dimensional electrons, we have depicted how to use our result in order to study the back-scattering problem by means of the non-local sine-Gordon theory proposed in this paper. We have also stressed the possibility of using our result as an alternative tool to check the validity of Coleman's equivalence at finite volume.

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